Design and Analysis of Information Systems

How to make the best strategy

Sure win in Casino

- You go to Casino and the dealer reveals 52 cards one by one

- You have 1000 dollars, and bet to red or blue 52 times. Minimum bet is 1 dollar, and the maximum is 10,000 dollars.

- How much you can gain (surely?)

- If you bet (say, 20 dollars) each time randomly, no sure gain.

Easy solutions

- Counting cards
  - Wait until the last card counting previous cards, then you know the color of last card.
  - Then, bet 1000 dollars.   You have 2000 dollars.
  - Are you happy with 1000 win?
- Wait until the last three cards. Suppose 2 red and 1 blue remain.
  - Bet 1000/3 dollars on red (in practice, 333)
  - If you win, wait until last, and you will have 8000/3
  - If you lose, bet all to red twice to have 8000/3
  - You gain about 1666
- Is this best? What is your idea???

If no upper limit

- Gambler's strategy
  - You assume one of possible sequences of red-blue, and bet all believing your luck.
  - If you are lucky, you gain 2^{52} x 1000 dollars
- Does not make sense? But expectation is good, since there are only $_{2n}C_n$ possible sequences. $$_{2n}C_n \approx 2^{2n}/\sqrt{\pi n}$$
- Expectation is about $\sqrt{26\pi} \times 1000 \approx 9000$
- However, there are two defects
  - Very low probability to obtain gain
  - Upper limit of bet prevent this strategy

Game theory

- Pure strategy: Do something determnistic
- Mixed strategy: Affine linear combination of pure strategies.
  - Randomized, but often we convert it to deterministic
- You gather all gamblers, and run in parallel
  - Is it possible?
  - This is what economists want to do….

100 boxes

- Cruel king wants to play with 100 prisoners.
- He places 100 boxes on a long table, and place names of prisoners (one in each box)
- Each prisoner is called one by one, and opens at most 50 boxes. If he finds his name, "success".
- If all prisoner success, King will release all.
  - Otherwise, all are kept in prison.
- King plays this game every Sunday, and execute them after 10 weeks (if not released)
- Prisoners can discuss before each game. But no communication is allowed once the game starts.