Smart Workshop

Exploring Collaborative Mathematics

- **Date:**
  March 13-14, 2012
- **Venue:**
  Large Lecture Hall (2F), Graduate School of Information Sciences, Tohoku University
- **Organizing Committee:**
  Nobuaki Obata, Motoko Kotani, Keiji Miura, Michiaki Onodera, Masaya Maeda
- **Sponsored by**
  Tohoku University's Focused Research Project
  “Interdisciplinary Mathematics Toward Smart Innovations”

Program

March 13 (Tuesday)

10:00-10:20 Takehisa Hasegawa (Graduate School of Information Sciences, Tohoku Univ.)
Novel percolation transition in complex networks and nonamenable graphs

10:20-10:40 Keiji Miura (Graduate School of Information Sciences, Tohoku Univ.)
An unbiased estimator of noise correlations under signal drift

11:00-11:20 Hirotoshi Kuroda (Graduate School of Science, Tohoku Univ.)
The density of state of the Tomonaga-Luttinger liquid with geometric conditions

11:20-11:40 Kaname Matsue (Graduate School of Science, Tohoku Univ.)
Rigorous numerical verification of local dynamics around equilibria of dynamics in infinite dimensions

<Lunch>

13:10 Michitaka Kameyama (Graduate School of Information Sciences, Dean)
Welcome Address

13:20-13:40 Masaya Maeda (Graduate School of Science, Tohoku Univ.)
Concentration of the least energy solution in a thin domain

13:40-14:00 Michiaki Onodera (Graduate School of Science, Tohoku Univ.)
A new geometric flow of surfaces and its applications

14:00-14:20 Kanako Suzuki (Graduate School of Information Sciences, Tohoku Univ.)
Patterns in activator-inhibitor systems based on the diffusion driven instability

<Short break>

14:45-15:35 Uzy Similansky (Weizmann Institute of Physical Sciences, Israel)
Oscillations and Vibrations — The Sturm and Courant Theorems Revisited

<Short break>

16:00-16:50 Yoshitaka Kimura (International Advanced Research and Education Organization, Tohoku Univ.)
Mathmatical formulars of gene-protein netowrk

March 14 (Wednesday)

10:00-10:30 Etsuo Segawa (Graduate School of Information Science and Technology, Tokyo Univ.)
Brief invitation to quantum walks

<Short break>

11:00-11:20 Ryokichi Tanaka (WPI-AIMR Tohoku Univ.)
Mendel-Naor's ultrametric subsets and geometric data analysis

11:20-11:40 Natsuto Yoshinaga (WPI-AIMR Tohoku Univ.)
Self-propulsion of a drop driven by Marangoni flow

<Lunch>

13:30-14:20 Hiroya Nakao (Graduate School of Information Science and Engineering, Tokyo Institute of Technology)
Spatio-temporal pattern formation in reaction-diffusion systems on complex networks

The 26th AMF Workshop

15:00-15:50 Arvind Kumar (University of Freiburg, Germany)
The critical role of striatal inhibition in shaping the oscillatory activity in the basal ganglia

16:10-17:00 Uzy Smilansky (Weizmann Institute of Physical Sciences, Israel)
Nonlinear Schroedinger Equation on Networks

Abstracts

Novel percolation transition in complex networks and nonamenable graphs

Takehisa Hasegawa (Graduate School of Information Sciences, Tohoku Univ.)

We study phase transition of bond percolation on a nonamenable graph (NAG) and several network models. The phase transitions of these networked systems are quite new: these systems exhibit the critical phase, which is never observed on the Euclidean lattices. First, we investigate bond percolation on the enhanced binary tree (EBT), which is given by adding intra-generation links to the binary tree, as an example of NAG. Here NAG is defined to be a graph with positive Cheeger constant. For bond percolation on a NAG, three phases appear according to open bond probability p: (i) nonpercolating phase where there is no infinite cluster for $0 < p < p_c1$, (ii) critical phase where there are infinitely many infinite clusters for $p_c1 < p < p_c2$, and (iii) percolating phase where there is a unique infinite cluster for $p_c1 < p < 1$. We perform Monte-Carlo simulations to give a physical picture. We also consider bond percolation on complex networks. Critical phenomena of bond percolation on complex networks crucially depend on their network topology. In particular, some network models under growth mechanisms show the critical phase. We investigate the phase diagram of bond percolation on several networks via Monte-Carlo simulation and renormalization group technique.

An unbiased estimator of noise correlations under signal drift

Keiji Miura (Graduate School of Information Sciences, Tohoku Univ.)

It is hard to estimate noise correlations from nonstationary multivariate time series because the conventional correlogram is contaminated by time-dependent signals (means), which are typically unknown. Here I derive an estimator of covariance matrix of Gaussian noises which work whatever the time course of the signal is.

The density of state of the Tomonaga-Luttinger liquid with geometric conditions

Hirotoshi Kuroda (Graduate School of Science, Tohoku Univ.)

A Tomonaga-Luttinger liquid (TLL) is a theoretical model which describes interacting electrons in one-dimensional case. It is well known that Fermi liquid theory breaks down for TLL model and power-law type singularities appear in the density of state, because interactions have drastic effects compared to higher dimension. In this talk we consider this model on a thin two-dimensional tubular surface with periodic radius modulation to study the effects of geometric conditions.

Rigorous numerical verification of local dynamics around equilibria of dynamics in infinite dimensions

Kaname Matsue (Graduate School of Science, Tohoku Univ.)

We briefly show a criterion for analyzing local dynamics of equilibria of dissipative PDEs. Our method can be applied to various parabolic PDEs with computer assistance, and this approach will be extended to analyze further dynamical information of PDEs such as time-periodic orbits or connecting orbits.

Concentration of the least energy solution in a thin domain

Masaya Maeda (Graduate School of Science, Tohoku Univ.)

We consider a nonlinear elliptic equation with small diffusion term in a thin domain with a Neumann boundary condition. It is shown that the least energy solution of the elliptic equation concentrates on a line in the domain. This is contrast to the case of ordinary domain which the least energy solution concentrates at a single point on the boundary of the domain.

A new geometric flow of surfaces and its applications

Michiaki Onodera (Graduate School of Science, Tohoku Univ.)

Hele-Shaw flow is an incompressible viscous fluid flow in an experimental device which consists of two closely placed parallel plates. An interesting feature of Hele-Shaw flow is that the evolution of a fluid domain under the flow produced by injection of fluid does not change its geometric moments in time, while the area increases. We will introduce a new geometric flow which has an analogous property, that is, the moments of the boundary of the domain are preserved under the flow. Applications to other problems will also be mentioned.

Patterns in activator-inhibitor systems based on the diffusion driven instability

Kanako Suzuki (Graduate School of Information Sciences, Tohoku Univ.)

We consider non-homogeneous stationary solutions of an activator-inhibitor system which has the diffusion-driven instability. In the system, it is assumed that the inhibitor diffuses much faster than the activator. It is expected that stable spatial patterns emerge, but all non-homogeneous stationary solutions can be unstable if the activator does not diffuses. We can see the importance of diffusion to obtain stable spatial patterns.

Oscillations and Vibrations --- The Sturm and Courant Theorems Revisited

Uzy Similansky (Weizmann Institute of Physical Sciences, Israel)

Consider the Laplace (Schrödinger) operator on a domain $\Omega\subset R^d$. Dirichlet boundary conditions are assumed on $\partial\Omega$. Arrange the spectrum as a non-decreasing sequence and consider the *n*'th eigenfunction $f^{(n)}$. The *number of nodal domains* ν_n is the number of maximally connected subdomains where the eigenfunction has a constant sign. For *d=1* Sturm's oscillation theory states that the number of sign changes of the wave function ϕ_n =n-1 and consequently $n=\nu_n$. For *d>1* ϕ_n is not defined, and Courant's theorem states that $n\geq\nu_n$. In the present talk I shall discuss the *nodal deficiency* = $n-\nu$, and show that it contains valuable information on the geometry of $\partial\Omega$. In particular I shall present recent results which show that nodal deficiency can be derived from a new variational approach, and that it is equal to the Morse index (the number of unstable directions) of a properly defined Energy functional at its critical points. The generalization of Sturm's oscillation theorem will be presented for the discrete Schrödinger operator on graphs, where the variational principle mentioned above takes a simple form.

Mathmatical formulars of gene-protein netowrk

Yoshitaka Kimura (International Advanced Research and Education Organization, Tohoku Univ.)

細胞内で繰り広げられる多因子系の化学反応には、核内でおきる **DNA** から **mRNA** への転写と核外の細胞質で起きる **mRNA** から蛋白質への変化、および、蛋白質同士の相互作用ネットワークがある。これらは、一部核内に移行し転写因子として核内の巨大蛋白を形成し遺伝子発現そのものを制御したり、細胞質で酵素反応をおこし、代謝産物を生産し、細胞構造を変化させたり細胞壁から分泌したりする。この時起きる種々の反応の時間変化の特徴は、作用主体である遺伝子、蛋白ネットワークと生成産物である代謝産物分泌のネットワークにおいて決定的に違う構造が考えられる。ここでは、これらの時間構造の違いからカオスの縁としての遺伝子、蛋白ネットワークの実態に迫る。

Brief invitation to quantum walks

Etsuo Segawa (Graduate School of Information Science and Technology, Tokyo Univ.)

Since high efficiency of quantum walks to quantum speed up algorithms was shown around 2000, nowadays the quantum walk has been intensively studied by many researchers from various viewpoints: for example, fundamental physics, spectral analysis on the unit circle, stochastic behavior comparing with its corresponding random walk, the Anderson localization, and photosynthesis as its mathematical model. In this talk, we briefly review on the above works and two important stochastic properties of the quantum walk i.e., ballistic spreading and localization, which are considered to ensure its nice effectiveness of applications.

Mendel-Naor's ultrametric subsets and geometric data analysis

Ryokichi Tanaka (WPI-AIMR, Tohoku Univ.)

Mendel and Naor show that, given the distortion bound, for any compact metric space there exists a subset which can be embedded into an ultrametric space with “large” Hausdorff dimension. I introduce their results and discuss with relation to data analysis.

Self-propulsion of a drop driven by Marangoni flow

Natsuto Yoshinaga (WPI-AIMR, Tohoku Univ.)

**Spontaneous motion or self-propulsion have been attracting attention in last decades for its potential application to biological problems such as cell motility and wound healing. Recently several model experiments showing spontaneous motion have been proposed. The systems
in these works consist of relatively simple ingredients for instance oil drops in water nevertheless the motion is as if the drops are alive. The key questions are why the particle moves without external force and why it breaks symmetry and chooses one direction. The first point has been discussed in hydrodynamics of the Marangoni effect in which a liquid droplet is driven by a gradient of surface tension. The mechanism is that the gradient induces convective flow inside and outside of a drop, which leads to swimming motion of the drop itself. The second point was less discussed, but has been discussed in the field of nonlinear dynamics as drift instability. Thus far there are only few attempts to discuss the spontaneous symmetry breaking from hydrodynamics. In this presentation, we show the nonlinear amplitude equations exhibiting drift instability, which can be derived from hydrodynamic and reaction-diffusion equations including the advection term.**

Spatio-temporal pattern formation in reaction-diffusion systems on complex networks

Hiroya Nakao (Graduate School of Information Science and Engineering, Tokyo Institute of Technology)

Spatio-temporal pattern formation in network-organized reaction-diffusion systems is investigated. The Turing instability in activator-inhibitor systems is a classical framework of non-equilibrium pattern formation in chemical and biological systems. Extending the past work by Othmer & Scriven (1971) on network Turing instability, we investigate Turing patterns on large scale-free networks with strong degree heterogeneity. The instability leads

to entirely different Turing patterns from those in ordinary continuous media. No spatially periodic patterns are observed, but rather the network nodes differentiate into distinct dynamical states depending on their degrees. The resulting patterns can qualitatively be understood by adopting the mean-field approximation of the network and considering the bifurcation diagrams of the individual nodes driven by the mean field. Similarly, coupled limit-cycle oscillators on scale-free networks can undergo Benjamin-Feir instability, which leads to high dimensional chaos on the network.

[1] H.Nakao & A.S.Mikhailov, Turing patterns in network-organized activator-inhibitor systems. Nature Phys. (2010), 544.
[2] H.Nakao & A.S.Mikhailov, Diffusion-induced instability and chaos in random oscillator networks. Phys. Rev. E 79 (2009), 036214.

The 26th AMF Workshop

The critical role of striatal inhibition in shaping the oscillatory activity in the basal ganglia

Arvind Kumar (University of Freiburg, Germany)

Movement disorders in Parkinson's disease (PD) are commonly associated with slow oscillations and increased synchrony of neuronal activity in the basal ganglia (subthalamic nucleus - STN, and globus pallidus external- GPe).
We investigated the dynamics of the basal ganglia using a reduced mean field model, which resembles the Lotka-Volterra equations used in modeling predator-prey relations in population biology. This model allowed us to isolate several biological plausible mechanism that could induce oscillations. Specifically, I will discuss how both firing rate and correlations among inhibitory inputs from the striatum to the GPe (i.e. increased inhibition of inhibitory network) control the oscillations in the basal ganglia. Consistent with experimental observations, we found that increase in either of these can unleash the oscillations in the basal ganglia, similar to that observed in the PD.
This observation allows us to propose a unified explanation for different phenomena: absence of oscillation in the healthy state of the basal ganglia, oscillations in dopamine-depleted state and quenching of oscillations under deep-brain-stimulation (DBS). Finally, studying the model behavior under transient increase of activity of the striatal neurons projecting to the indirect pathway, we are able to account for both motor impairment in PD patients and for reduced response inhibition in DBS implanted patients.

Nonlinear Schroedinger Equation on Networks

Uzy Smilansky (Weizmann Institute of Physical Sciences, Israel)

Transmission through a complex network of nonlinear one-dimensional leads is discussed by extending the stationary scattering theory on quantum graphs to the nonlinear regime. The resonances which dominate linear scattering are shown to be extremely sensitive to the nonlinearity and display multi-stability and hysteresis. This work provides a framework for the study of light propagation in complex optical networks, and for studying universal properties of Bose-Einstein Condensate (BEC) in connected chaotic traps.

東北大学における数学連携の取り組み

1) 応用数学連携フォーラム（平成 19 年 9 月～）
代表：尾畑、副代表：小谷、事務局：東北大学大学院情報科学研究科数学連携推進室
http://www.dais.is.tohoku.ac.jp/~amf/

2) CREST「離散幾何学から提案する新物質創成と物性発現の解明」
（平成 20～25 年度、研究代表者：小谷）
http://www.mathmate.tohoku.ac.jp/
科学技術振興機構 CREST 研究領域「数学と諸分野の協働によるブレークスルーの探索」
（領域総括：北海道大学電子科学研究所教授・西浦廉政）
http://www.math.jst.go.jp/

3) 東北大学重点戦略支援プログラム「数学をコアとするスマート・イノベーション融合研究共通基盤の構築と展開」（平成 22～26 年度、代表：尾畑）
http://www.dais.is.tohoku.ac.jp/~smart

4) 東北大学原子分子材料科学高等研究機構（WPI-AIMR）数学ユニット（平成 23 年度～）
http://www.wpi-aimr.tohoku.ac.jp/jp/

5) 東北大学大学院理学研究科数学専攻数学連携推進室（平成 23 年 1 月～）

6) 東北大学大学院情報科学研究科数学連携推進室（平成 23 年 1 月～）
http://www.is.tohoku.ac.jp/introduction/cmru/

応用数学連携フォーラム（AMF）へようこそ

フォーラムメンバー募集中！

数学は諸科学に共通する言語とも道具とも言われます。
数学は実にさまざま研究分野でいろいろな形で使われています。

応用数学連携フォーラムでは、数学と諸分野との連携に興味のある研究者たちが気軽に情報交換したり、研究交流したりするための場を提供しています。ちょっとのぞいてみたいという方は、随時開催しているワークショップなどはいかがでしょうか。院生や学部生の方も含めて、興味のある方はどなたでも自由にご参加いただけます。事前登録など必要ありませんので、どうぞ、お気軽にお越しください。詳しくは HP を見てください。