Ultimate Implementation and Analysis of the AMO Algorithm for Approximate Pricing of European-Asian Options

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(Tohoku University)

joint work with T. Tokuyama
Summary of This Talk

● PQUJPOɿUZQJDBMGJOBODJBMEFSJWBUJWF
● QSJDJOH&VSPQFBO"TJBOPQUJPO POCJOPNJBMNPEFM

EJGGJDVMUUPDPNQVUFBDDVSBUFMZ

B QQSPYJ

N

BUJ

P

O

JOHXPSUI

.PUXBOJ0MEIBN	40%

UJNF0	LO

BCTPMVUFFSSPS0	O9L

0VS"MHPSJUIN

UJNF0	LO

BCTPMVUFFSSPS0	9L

O

9QSPCMFNQBSBNFUFST
LUJNFFSSPSUSBEFPGGQBSBN
Option

- Option
- Option

- Option
- Option
- Option
Payoff of Option

- When the stock price is less than or equal to the strike price, the payoff is zero.
- When the stock price is greater than the strike price, the payoff is the stock price minus the strike price.

\[ S - X \]

(S: stock price, X: strike price)
European-Asian Option

- European-Asian Option

\[ v = \frac{1}{N} \sum_{i=1}^{N} S_i \]

\[ S_i \quad \text{strike price} \quad X \]

\[ UJNF \quad 4TUPDLQSJDF \quad BWFSBHFPG \quad TUPDLQSJDF \quad 49 \quad TBGFBJOTUGMVDUVBUJPOPGTUPDLQSJDF \]
Computation of Option Price

- The computation of option price is based on the Black-Scholes model.
- The model takes into account various factors such as stock price, strike price, time to expiration, risk-free rate, and volatility.
- The Black-Scholes formula provides a theoretical price for European call and put options.
- The model assumes constant volatility and no dividends on the underlying asset.
- However, in reality, these assumptions may not hold, leading to model risk.
- Despite its limitations, the Black-Scholes model remains a widely used tool in financial markets.
Binomial Model
Our Problem

\[ E \left( \left( \sum_{i=0}^{n} \frac{S_i}{n+1} - X \right)^+ \right) \]

- Some initial assumptions and observations (not fully visible)
- Additional points
- Further discussion
- Conclusion
Approximation Algorithms for Pricing European-Asian Option

- [ ] Section 1: Introduction
- [ ] Section 2: Preliminaries
- [ ] Section 3: Main Results
- [ ] Section 4: Conclusions
Exact Algorithm by DP

\[ \sum_{i=0}^{t} S_i \]
AMO Algorithm (1)

- # of running subtotals can be exponential
  - approximate running subtotals by bucketing

<table>
<thead>
<tr>
<th>interval</th>
<th>running subtotal &amp; probability</th>
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<tbody>
<tr>
<td>400 300</td>
<td>(310, 0.05)</td>
</tr>
<tr>
<td>300</td>
<td>(205, 0.15) (240, 0.12) (285, 0.20)</td>
</tr>
<tr>
<td>200</td>
<td>(170, 0.10) (150, 0.10) (110, 0.10)</td>
</tr>
<tr>
<td>100</td>
<td>(80, 0.05) (30, 0.01)</td>
</tr>
<tr>
<td>100 0</td>
<td>(400, 0.05) (300, 0.47) (200, 0.30) (100, 0.06)</td>
</tr>
</tbody>
</table>
AMO Algorithm (2)

- The AMO Algorithm for Image Segmentation

\[ \sum_{i=0}^{t} S_i \]

- Segmentation
- Sub-image
- Image

- Summary of the algorithm:
  - Step 1: Initialize
  - Step 2: Process
  - Step 3: Output

- The AMO Algorithm in practice
Algorithm by Dai et al. (2002)

\[ \sum_{i=0}^{n} \sum_{j=0}^{i} \frac{X}{k_{ij}} \omega(i, j) \]

\[ O\left(\frac{\sqrt{nX}}{k}\right) \]
Algorithm by Ohta et al. (2002)

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- 200 0.60
- 170 0.60
- 150 0.60
- 110 0.60
Analysis of Ohta et al. (2002)

- Error bound: $O\left(\frac{1}{n^4} \frac{X}{k}\right)$ (with high probability)
Our Algorithm

- $\sum_{i=1}^{n} \sum_{j=0}^{i} \left[ \frac{\omega(i, j)}{k_{ij}} \right]^2$

- $O\left( \frac{X}{k} \right)$
Open Problems

- derandomization of our algorithm with the same error bound
- approximation of American-Asian option
- analysis of error bound compared to exact price