PAPER

A Unified View to Greedy Geometric Routing Algorithms in Ad Hoc Networks

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SUMMARY We give a unified view to greedy geometric routing algorithms in ad hoc networks. For this, we first present a general form of greedy routing algorithm using a class of objective functions which are invariant under congruent transformations of a point set. We show that several known greedy routing algorithms such as Greedy Routing, Compass Routing, and Midpoint Routing can be regarded as special cases of the generalized greedy routing algorithm. In addition, inspired by the unified view of greedy routing, we propose three new greedy routing algorithms. We then derive a sufficient condition for our generalized greedy routing algorithm to guarantee packet delivery on every Delaunay graph. This condition makes it easier to check whether a given routing algorithm guarantees packet delivery, and it is closed under convex linear combination of objective functions. It is shown that Greedy Routing, Midpoint Routing, and the three new greedy routing algorithms proposed in this paper satisfy the sufficient condition, i.e., they guarantee packet delivery on Delaunay graphs. We also discuss merits and demerits of these methods.
key words: *geometric routing, ad hoc network, greedy routing, Delaunay graph*

1. Introduction

Ad hoc network is an autonomous system that does not require a pre-established infrastructure. Nodes in ad hoc networks are connected by wireless links, and the communications between nodes are often achieved by multi-hop links. With increased interests in mobile communications and the promise of convenient infrastructure-free communications, the development of large-scale ad hoc networks has drawn a lot of attention and has been a subject of extensive research.

Geometric routing (also called geographic routing or position-based routing) is a technique in ad hoc networks to send a packet (or a message) from a source node to a destination node by repeatedly forwarding a packet to an appropriately chosen neighbor node. Geometric routing finds a route by using the location of the destination node and local location information, i.e., the locations of a current node and its neighbors, and does not require the knowledge of the entire network. See, e.g., [8], [11] for detailed survey of geometric routing algorithms.

The first approaches for geometric routing algorithms were developed in the 1980s, and they are based on greedy strategies (see, e.g., [3], [12]); that is, they repeatedly forward a packet to a neighbor which is "closer" to the destination node than other neighbors with respect to various criteria of "closeness." Since then, various greedy routing algorithms have been proposed in the literature.

One of the most popular and natural greedy routing algorithm is Greedy Routing by Finn [3], which forwards a packet to a neighbor with the minimum Euclidean distance to the destination. Another popular routing algorithm is Compass Routing by Krankakis, Singh, and Urrutia [4]. The idea of Compass Routing is to forward a packet to a neighbor with the minimum angle $\angle wvt$, where v is a current node, w is a neighbor, and t is the destination. A greedy routing algorithm of a different flavor is Midpoint Routing proposed by Si and Zomaya [10]. The idea of Midpoint Routing is to find a neighbor of a current node which minimizes the Euclidean distance to the midpoint of the current node and the destination. These three greedy algorithms are discussed in this paper.

The main aim of this paper is to give a unified view to existing greedy routing algorithms. For this, we first present a general form of greedy routing algorithm by using a general objective function f defined on triplets of nodes; the value $f(w, v, t)$ of the objective function f depends only on three nodes v, w and t corresponding to current node, its neighbor node, and the destination node, respectively. The generalized greedy routing algorithm repeatedly forwards a packet to a neighbor node minimizing the function value of f. In particular, we introduce a class of objective functions, called *congruence-invariant objective functions*, which satisfy the condition $f(w, v, t) = f(w', v', t')$ whenever the two triangles $\triangle wvt$ and $\triangle w'v't'$ are *congruent*, i.e., $\triangle w'v't'$ can be obtained from $\triangle wvt$ by a combination of translations, rotations, and reflections. We show that various existing routing algorithms such as Greedy Routing, Compass Routing, and Midpoint Routing can be regarded as special cases of the generalized greedy routing algorithm with congruence-invariant objective functions. Moreover, inspired by the unified view of greedy routing, we propose three new greedy routing algorithms with congruence-invariant objective functions.

One of the most important factors of routing algorithms is *guaranteed delivery* of packets. In this paper, we discuss in which situation (and by which routing algorithm) packet delivery from a given source to a given destination is guaranteed. In general, greedy routing algorithms often fail to deliver a packet to the destination due to the existence of a *local minimum*; local minimum is a node which has no neighbor closer to the destination. On the other hand, it is shown that some greedy routing algorithms succeed in deliv-

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ery of packets if the network topology has a nice structure. We can represent the topology of an ad hoc network by using an undirected graph $G = (P, E)$ defined on the node set P, where E is the set of all pairs of two distinct nodes in P which can communicate to each other. It is shown that Greedy Routing, Compass Routing, and Midpoint Routing always guarantee packet delivery if G is the Delaunay graph of the node set P (see Section 2 for the definition of Delaunay graph), although the proofs of the statements are given independently in an ad hoc manner in [1], [4], [10].

Aiming at providing a unified view to greedy routing algorithms with guaranteed delivery, we derive a sufficient condition for our generalized greedy routing algorithm to guarantee packet delivery on Delaunay graphs. This condition makes it easier to check whether a given routing algorithm guarantees packet delivery. Indeed, we show that Greedy Routing, Midpoint Routing, and the three new greedy routing algorithms proposed in this paper satisfy the sufficient condition. This provides alternative proofs for Greedy Routing and Midpoint Routing, while this implies that the new routing algorithms guarantee packet delivery on Delaunay graphs. Delaunay graph has a nice property as Euclidean spanner [2], which assures a small ratio of the shortest-path length on it to the Euclidean distance, and it can be constructed in a distributed fashion. Thus, it is a popular network in ad hoc network design. Moreover, we can show a stronger statement that these routing algorithms work on any supergraph of the Delaunay graph of a given node set. Our new algorithms are simple and have nice properties that were not assured in previous ones. We also give an algebraic property on a family of algorithms which makes it possible to obtain hybrid algorithms systematically.

The organization of this paper is as follows. Section 2 is devoted to preliminaries on fundamental concepts used in this paper. A generalized greedy routing algorithm is proposed in Section 3, and guaranteed delivery of the algorithm on Delaunay graphs is shown in Section 4. Discussion on how to design ad hoc networks with efficient geometric routing algorithms is given in Section 5. Greedy routing algorithms considered in this paper are compared theoretically and experimentally in Section 6. Finally, concluding remarks are given in Section 7.

2. Preliminaries

2.1 Ad hoc networks

Ad hoc network is a decentralized type of wireless network made up of multiple nodes (devices) connected by wireless links. One common feature of ad hoc network is that it is constantly changing. The popular type of ad hoc network is mobile ad hoc network (MANET), in which devices are allowed to move freely in any direction, which makes some of the connections broken and some new links formed. Therefore, links between one device and others change frequently and it is necessary for every device to maintain continuously the information of itself and its neighbors. Ad hoc network, in general, has no base station, that makes the scalability of the network better compared to fixed networks. Two important properties of geometric routings in ad hoc network are transmission time and power consumption. Transmission time, which is an important property in all networks, determines how fast a device can send a packet to another, and it is roughly proportional to the number of hops required to send packet from the source to the destination node. Energy consumption is also important, especially for networks of small devices of limited power. If the devices have unlimited power, the problem becomes trivial, because we can send as much data as we want from one device to another regardless of distance between them. Unfortunately, problem of minimizing transmission time and that of minimizing power consumption are incompatible. There are some tradeoffs between these two properties based on requirements of applications, making designing communication protocols for ad hoc network even more challenging.

2.2 Model of ad hoc networks

Let P be a finite set of nodes on the Euclidean plane. Our problem is to find a route from a given source node s to a given destination node t in the ad hoc network on P. In the following discussion, we often represent the topology of the network by an undirected graph $G = (P, E)$ on P, where E is the set of all pairs of two distinct nodes in P which can communicate to each other. We say that a node w is a *neighbor* of another node v if w and v are connected by an edge.

Although our results hold independent of choice of models of ad hoc networks, we give some remarks to help understanding of readers. The graph structure G is given in a way that each node has the location information of its neighbors in G. Each node can send information to nodes in its transmission radius, and we assume that each node can control the transmission radius so that it can send packet to a neighbor node by setting the radius to be larger than the distance to the neighbor. Note that there is another model in which each node has a fixed common transmission radius, called *unit disk model*; the corresponding graph structure is called a *unit disk graph*. We describe algorithms in the case where the point set P and graph G are given in advance and static for simplicity. We also consider the mobile ad hoc network in which the points in P may move and G is dynamically updated, since geometric routing is particularly useful in the mobile ad hoc network where it is expensive to maintain global routing table at each node. We will discuss mobile ad hoc networks when we compare algorithms in Section 5.

The location of the destination t is assumed to be given by some mechanism, and we do not consider the issue of how to obtain the location of the destination in this paper. For example, one may assume a model in which the destination is selected from a set of static nodes (called *hubs*) whose positions are known to all nodes, and communication will be done via such hubs.

2.3 Geometric preliminaries

For simplicity we assume, throughout this paper, that nodes in P are in general position. This implies, in particular, that there are no four nodes which lie on the same circle. This assumption can be removed by using symbolic perturbation.

Voronoi diagram of P is a partition of the Euclidean plane $\mathbb{R}^2$ into convex polygonal cells corresponding to points in P such that all points in a cell corresponding to $v \in P$ are closer to v than other nodes in P. *Delaunay triangulation* of P is a triangulation of P such that two nodes $u, v \in P$ are connected by a straight line if and only if two cells corresponding to u and v have a common edge in the Voronoi diagram of P. We regard a Delaunay triangulation as an undirected graph $G = (P, E)$ on P, and call G a *Delaunay graph* of P.

For every nodes $x, y \in \mathbb{R}^2$, we denote by $d(x, y)$ the Euclidean distance between x and y, i.e., $d(x, y) = \|x - y\|_2$. A *closed disk* $D \subseteq \mathbb{R}^2$ is a set of points in $\mathbb{R}^2$ given as $D = \{x \in \mathbb{R}^2 \mid d(x, c) \leq \lambda\}$ for some $c \in \mathbb{R}^2$ and $\lambda > 0$. The interior of D is an open set given as $\{x \in \mathbb{R}^2 \mid d(x, c) < \lambda\}$. The following property is known as a folklore (see, e.g., [6, Theorem 9.6]).

Proposition 1. *In the Delaunay graph $G = (P, E)$ of P, two nodes $u, v \in P$ are adjacent to each other if and only if there exists a closed disk $D \subseteq \mathbb{R}^2$ such that u and v lie on the boundary of D and any other node is not contained in the interior of D.*

For a pair (u, v) of points of P, the *Gabriel disk* of (u, v) is defined as the (unique) disk $D(u, v)$ that has the line segment connecting u and v as its diameter chord. If the Gabriel disk $D(u, v)$ is empty, i.e., contains no point of P other than u and v, then (u, v) is called an *Gabriel edge*. The graph on the vertex set P with Gabriel edges is called the *Gabriel graph* on P. Proposition 1 implies that every Gabriel edge is an edge of Delaunay graph; thus, the Gabriel graph is a subgraph of a Delaunay graph. We often say that G is a *supergraph* of G' if G' is a subgraph of G.

3. Greedy Routing Using General Objective Functions

Denote by T the set of triplets of distinct three nodes, i.e.,

$$T = \{(w, v, t) \mid w, v, t \text{ are distinct nodes in } P\}.$$

It is noted that each element $(w, v, t) \in T$ is an ordered set, i.e., (w, v, t) and (v, t, w) are different elements.

We propose a greedy routing algorithm using a general objective function $f : T \to \mathbb{R} \cup \{+\infty\}$. This routing algorithm repeatedly forwards a packet to some neighbor w of a current node v which minimizes the function value $f(w, v, t)$ among all neighbors of v until a packet reaches the destination t. The routing algorithm is described as follows.

Algorithm Generalized Greedy Routing
Step 0: Set $v := s$.
Step 1: If t is a neighbor of v in G, then set $v := t$ (i.e., forward a packet to t) and stop.
Step 2: Select a neighbor w of v in G which minimizes the value $f(w, v, t)$ among all neighbors of v, and set $v := w$ (i.e., forward a packet to w). Go to Step 1.

We consider restricted classes of objective functions in Generalized Greedy Routing. We say that an objective function $f : T \to \mathbb{R} \cup \{+\infty\}$ is *congruence-invariant* if the function value $f(w, v, t)$ depends only on the shape and the size of the triangle $\triangle wvt$ given by three nodes w, v, and t. More precisely, a function $f : T \to \mathbb{R} \cup \{+\infty\}$ is said to be congruence-invariant if there exists a function $h : \mathbb{R}^6_+ \to \mathbb{R} \cup \{+\infty\}$ such that

$$f(w, v, t) = h(d_{vt}, d_{wt}, d_{vw}, a_t, a_w, a_v) \qquad ((w, v, t) \in T),$$

where each parameter in h is given as follows:

$$d_{vt} = d(v, t),\ d_{wt} = d(w, t),\ d_{vw} = d(v, w),$$
$$a_t = \angle vtw,\ a_w = \angle twv,\ a_v = \angle wvt.$$

It is noted that the parameters $d_{vt}, d_{wt}, d_{vw}, a_t, a_w, a_v$ used in the function h are dependent; for example, $a_t + a_w + a_v = \pi$ holds. We keep this redundancy for simplicity of presentation. It is easy to see that the function values of a congruence-invariant objective function do not change even if we transform the given node set by a combination of translations, rotations, and reflections, i.e., the two node sets before and after the transformation are congruent.

In the following, we mainly discuss the algorithm Generalized Greedy Routing with a congruence-invariant objective function. Many existing greedy routing algorithms can be represented as a special case of Generalized Greedy Routing by using appropriate congruence-invariant objective functions, as shown below.

Greedy Routing [3]: It chooses a neighbor w with the minimum distance $d(w, t)$. The corresponding congruence-invariant function is given as

$$f_{\mathrm{G}}(w, v, t) = h_{\mathrm{G}}(d_{vt}, d_{wt}, d_{vw}, a_t, a_w, a_v) = d_{wt}.$$

Compass Routing [4]: It chooses a neighbor w with the minimum angle $\angle tvw$. The corresponding congruence-invariant function is given as

$$f_{\mathrm{C}}(w, v, t) = h_{\mathrm{C}}(d_{vt}, d_{wt}, d_{vw}, a_t, a_w, a_v) = a_v.$$

Midpoint Routing [10]: It chooses a neighbor w with the minimum distance between w and $(v + t)/2$, the midpoint of v and t. The corresponding congruence-invariant function is given as

$$\begin{aligned} f_{\mathrm{MP}}(w, v, t) &= h_{\mathrm{MP}}(d_{vt}, d_{wt}, d_{vw}, a_t, a_w, a_v) \\ &= (d_{wt} \sin a_t)^2 + (d_{wt} \cos a_t - (1/2) d_{vt})^2. \end{aligned}$$

Modified Midpoint Routing [10]: This algorithm is considered in [10] as a generalization of Midpoint Routing. Given a fixed real number $\lambda \in [0.5, 1]$, it chooses a neighbor w with the minimum distance between w and

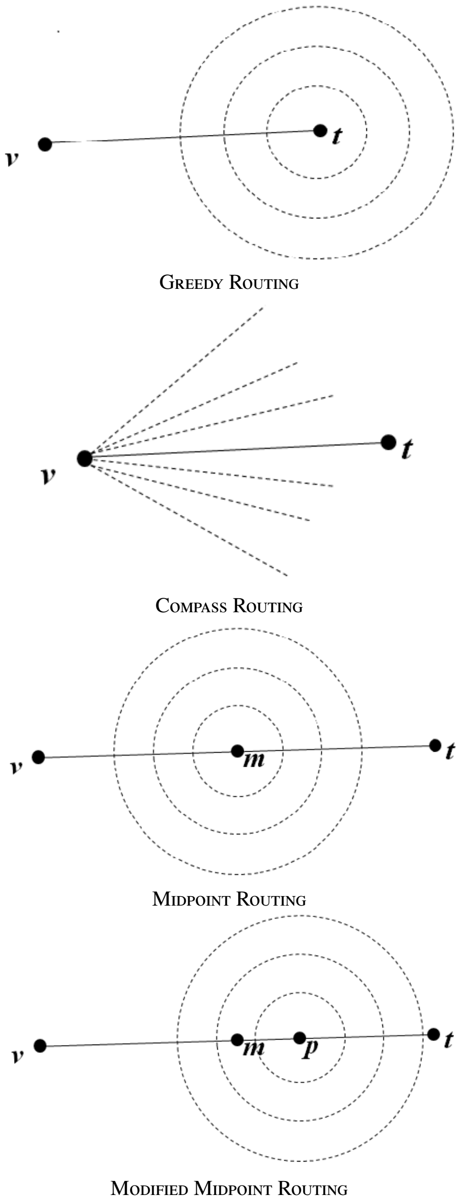

Fig. 1 Contour maps of congruence-invariant functions.

$(1-\lambda)v+\lambda t$. If $\lambda = 1$ (resp., $\lambda = 1/2$), then MODIFIED MIDPOINT ROUTING coincides with GREEDY ROUTING (resp., MIDPOINT ROUTING). The corresponding congruence-invariant function is given as

$$\begin{aligned} f_{\mathrm{MMP}}(w,v,t) &= h_{\mathrm{MMP}}(d_{vt}, d_{wt}, d_{vw}, a_t, a_w, a_v) \\ &= (d_{wt}\sin a_t)^2 + (d_{wt}\cos a_t - \lambda d_{vt})^2. \end{aligned}$$

Contour maps of congruence-invariant functions of those algorithms are shown in Figure 1.

In addition, we propose three new greedy routing algorithms, each of which can be represented as a special case of GENERALIZED GREEDY ROUTING by using some congruence-invariant objective functions. Note that the names of the new algorithms come from the contour maps of the corresponding objective functions (see Figure 2). For $\alpha \in \mathbb{R}$, let

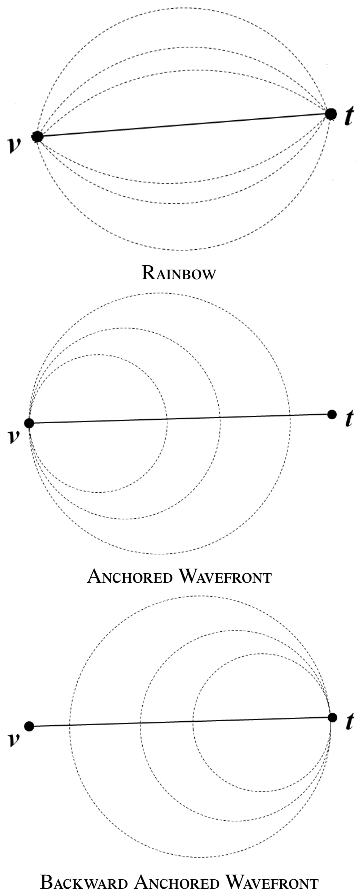

Fig. 2 Contour maps of new congruence-invariant functions.

$\varphi_\alpha : \mathbb{R} \to \{0, +\infty\}$ be a function given by

$$\varphi_\alpha(\beta) = \begin{cases} 0 & (\text{if } \beta < \alpha), \\ +\infty & (\text{otherwise}). \end{cases}$$

RAINBOW: This is a variant of COMPASS ROUTING, and chooses a neighbor w with the *maximum* angle $\angle vwt$. The corresponding congruence-invariant function is given as

$$f_1(w,v,t) = h_1(d_{vt}, d_{wt}, d_{vw}, a_t, a_w, a_v) = -a_w.$$

ANCHORED WAVEFRONT: This can be seen as a combination of COMPASS ROUTING and GREEDY ROUTING, and it chooses a neighbor w with the minimum value of $d(v,w)/\cos(\angle tvw)$ under the condition that $\angle tvw < \pi/2$. The corresponding congruence-invariant function is given as

$$\begin{aligned} f_2(w,v,t) &= h_2(d_{vt}, d_{wt}, d_{vw}, a_t, a_w, a_v) \\ &= \frac{d_{vw}}{\cos a_v} + \varphi_{\pi/2}(a_v). \end{aligned}$$

BACKWARD ANCHORED WAVEFRONT: This can be also seen as a combination of COMPASS ROUTING and GREEDY ROUTING, and it chooses a neighbor w with the minimum value of $d(w,t)/cos(\angle wtv)$ under the condition that $\angle wtv < \pi/2$. The corresponding congruence-invariant function is given as

$$\begin{aligned} f_3(w, v, t) &= h_3(d_{vt}, d_{wt}, d_{vw}, a_t, a_w, a_v) \\ &= \frac{d_{wt}}{\cos a_t} + \varphi_{\pi/2}(a_t). \end{aligned}$$

To give visual intuition of ideas, contour maps of congruence-invariant functions of algorithms are given in Figure 2. Each of the proposed algorithm has an advantage to known algorithms as shown later in Section 6.

4. Guaranteed Delivery on Delaunay Graphs

As mentioned in Introduction, all of the four algorithms Greedy Routing, Compass Routing, Midpoint Routing, and Modified Midpoint Routing, each of which is a special case of Generalized Greedy Routing, always guarantee packet delivery on Delaunay graphs. In this section, we derive a sufficient condition for Generalized Greedy Routing to guarantee packet delivery on Delaunay graphs.

Lemma 2. *Let G be the Delaunay graph of P and $u, v \in P$ be distinct nodes of G. Suppose that u and v lie on the boundary of some closed disk $D \subseteq \mathbb{R}^2$ and u is not a neighbor of v in G. Then, there exists a neighbor w of v in G such that w is in the interior of D.*

Proof. By Proposition 1, there exists some $w \in P$ contained in the interior of D since u and v are not adjacent. Let $w_1, w_2, \ldots, w_k \in P$ be the nodes contained in the interior of D. For each $i = 1, 2, \ldots, k$, let D_i be the closed disk such that w_i and v lie on the boundary of D_i and (the boundary of) D_i is tangent to (the boundary of) D at v. Assume, without loss of generality, that the radius of D_i is smaller than or equal to that of D_{i+1} for $i = 1, 2, \ldots, k-1$. Then, D_1 does not contain any nodes of $P \setminus \{v, w_1\}$ in its interior. Hence, w_1 is a neighbor of v in G by Proposition 1. □

Consider the following DDG (Delaunay Delivery Guarantee) condition for an objective function f. For a closed disk D, we denote by int$D$ the interior of D.

(DDG) For every distinct nodes $w, v, t \in P$, if

$$f(w, v, t) \le \max\{f(u, v, t) \mid u \in (P \setminus \{v, t\}) \cap \mathrm{int}D(v, t)\}, \quad (1)$$

then $d(w, t) < d(v, t)$ holds.

We explain this condition using Figure 3, where the dark disk is the Gabriel disk $D(v, t)$ of (v, t). The condition $d(w, t) < d(v, t)$ means that w is in the larger disk around t. Thus, DDG condition can be rephrased as follow: if the contour curve of f through the point w intersects the interior of the Gabriel disk, then w is in the larger disk.

We also consider the following stronger version of DDG condition:

(SDDG) For every distinct nodes $v, t \in P$, $f(u, v, t) < f(w, v, t)$ holds for every $w, u \in P \setminus \{v, t\}$ with $u \in \mathrm{int}D(v, t)$ and $w \notin \mathrm{int}D(v, t)$.

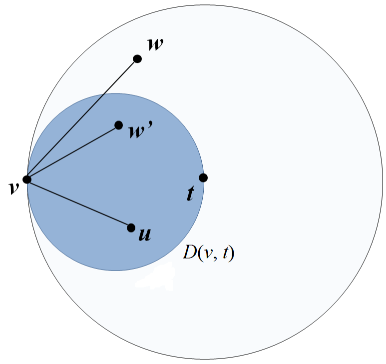

Fig. 3 DDG condition.

The SDDG condition can be rewritten as follows:

$$\begin{aligned} &\forall v, t \in P \text{ with } v \neq t: \\ &\max\{f(u, v, t) \mid u \in (P \setminus \{v, t\}) \cap \mathrm{int}D(v, t)\} \\ &\qquad < \min\{f(w, v, t) \mid w \in (P \setminus \{v, t\}) \setminus \mathrm{int}D(v, t)\}. \quad (2) \end{aligned}$$

SDDG condition implies DDG condition since there exists no node $w \in (P \setminus \{v, t\}) \setminus \mathrm{int}D(v, t)$ satisfying the condition (1). SDDG condition intuitively means that the contour curve of f grows in $D(v, t)$ and eventually coincides with its boundary circle, and then continues expanding to its outside.

The next theorem shows that DDG condition implies the guaranteed delivery of the algorithm Generalized Greedy Routing on Delaunay graphs and their supergraphs.

Theorem 3. *Let $f : T \to \mathbb{R} \cup \{+\infty\}$ be a function satisfying* DDG *condition. Then, the algorithm* Generalized Greedy Routing *with objective function f guarantees packet delivery on a supergraph G of the Delaunay graph of P.*

Proof. If t is adjacent to v, the algorithm certainly delivers the packet. Suppose that v is the current node which is not adjacent to the destination t, and w is the neighbor chosen by the algorithm. To prove that the algorithm guarantees packet delivery, it suffices to show the inequality $d(w, t) < d(v, t)$ since the number of nodes is finite.

Since v and t are non-adjacent in G, they are also non-adjacent in the Delaunay graph. Lemma 2 implies that there exists a neighbor u of v in G such that u is in the interior of the Gabriel disk $D(v, t)$ of (v, t) (see Figure 3). By the choice of w, we have $f(w, v, t) \le f(u, v, t)$. Hence, DDG condition implies $d(w, t) < d(v, t)$. □

One may wonder whether Generalized Greedy Routing satisfying DDG condition works if G is the Gabriel graph or its supergraph; this, however, is not true in general since there exists a counter-example as shown in Figure 4. While the Gabriel disk $D(v, t)$ is nonempty, the unique neighbor of v in the Gabriel graph is the point w_1 outside of $D(v, t)$. If we apply Greedy Routing, a packet is sent to w_1 from v, and w_1 sends back the packet to v since v is closer to t than w_2. Thus, Greedy Routing cannot send a packet to t on the Gabriel graph. In this connection, Khun et al. [5]

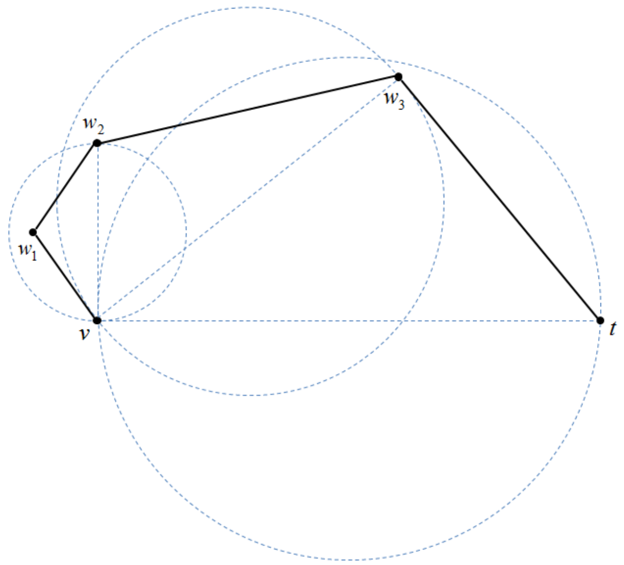

Fig. 4 A counter-example of Gabriel graph.

proposed a hybrid method that switches to the *face routing* if Greedy Routing gets stuck in order to guarantee packet delivery on the Gabriel graph.

We then show that the objective functions of Greedy Routing, Midpoint Routing, and Modified Midpoint Routing satisfy DDG condition. This fact provides us an alternative proof for the statement that these algorithms guarantee packet delivery on Delaunay graphs.

Lemma 4. *f_G, f_{MP}, and f_{MMP} satisfy* DDG *condition.*

Proof. Since f_G and f_{MP} are special cases of f_{MMP} with $\lambda = 1$ and $\lambda = 1/2$, respectively, we consider the function f_{MMP} only. Below we omit the subscript of f_{MMP} for simplicity.

Let w, v, t be distinct nodes, and assume that the condition (1) holds. We show that $d(w, t) < d(v, t)$ holds.

Let $p = (1 - \lambda)v + \lambda t$. Since $f(w, v, t) = d(w, p)$, the condition (1) can be rewritten as

$$d(w, p) \leq \max\{d(u, p) \mid u \in (P \setminus \{v, t\}) \cap \mathrm{int}D(v, t)\}.$$

It holds that

$$\begin{aligned} &\max\{d(u, p) \mid u \in (P \setminus \{v, t\}) \cap \mathrm{int}D(v, t)\}. \\ &< \sup\{d(x, p) \mid x \in D(v, t)\} \ = \ d(v, p). \end{aligned}$$

Hence, we have $d(w, p) < d(v, p)$. This implies that

$$d(w, t) \leq d(w, p) + d(p, t) < d(v, p) + d(p, t) = d(v, t).$$

□

It should be mentioned that among the algorithms above, only Midpoint Routing satisfies SDDG condition.

Theorem 5. *The algorithms* Greedy Routing, Midpoint Routing, *and* Modified Midpoint Routing *guarantee packet delivery on the Delaunay graphs and their supergraphs.*

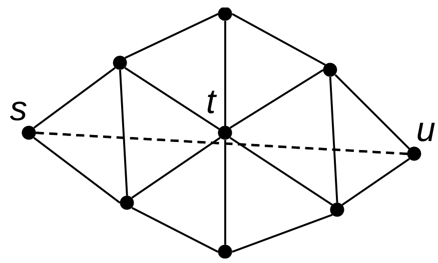

Fig. 5 An example for which Compass Routing fails.

Proof. The statement follows immediately from Theorem 3 and Lemma 4. □

We next show that our new routing algorithms also work on Delaunay graphs and their supergraphs. The following lemma immediately follows from the fact that contour curves grows inside the Gabriel disk, as easily observed from Figure 2.

Lemma 6. *f_1, f_2, and f_3 satisfy* SDDG *condition.*

This lemma, together with Theorem 3, implies the following:

Theorem 7. *The algorithms* Rainbow, Anchored Wavefront, *and* Backward Anchored Wavefront *guarantee packet delivery on any supergraph of the Delaunay graph.*

The property shown in Theorem 7 is an important advantage of the new algorithms when compared to Compass Routing, which does not guarantee packet delivery on supergraphs of Delaunay graphs.

Remark 8. The algorithm Compass Routing does not work on some supergraphs of Delaunay graphs, although it works on Delaunay graphs. For example, let G be the Delaunay graph shown in Figure 5 by solid edges, where t is the destination node. We consider a supergraph of G obtained by adding an edge (s, u) indicated by the dashed edge, and suppose that our current node is s. Then, the next node is u since the angle $\angle tsu$ is almost zero. In the next iteration, we come back to the node s since the angle $\angle tus$ is almost zero. Hence, we repeat moving between the two nodes s and u infinitely many times, and cannot reach to the destination t. □

Remark 9. It is noted that an objective function used in Generalized Greedy Routing should be carefully chosen to satisfy DDG condition. For example, the algorithm with the objective function $f(w, v, t) = d_{wt}^2 + d_{vw}^2$ is essentially the same as Midpoint Routing, and hence satisfies DDG condition. On the other hand, consider a similar objective function $f(w, v, t) = d_{wt} + d_{vw}$, for which the contour map consists of ellipses instead of circles. This objective function does not satisfy DDG; indeed, we can construct an example of node set for which Generalized Greedy Routing cannot deliver packets, as shown in Figure 6.

Figure 6 shows a node set and its associated Delaunay

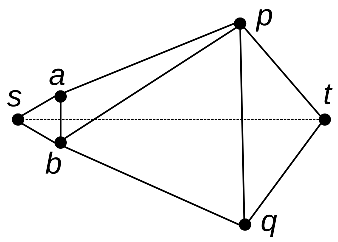

Fig. 6 A counter-example for GENERALIZED GREEDY ROUTING with the objective function $f(w, v) = d_{wt} + d_{vw}$.

graph. Note that nodes a and b are located symmetrically with respect to the line connecting s and t. Similarly, nodes p and q are located symmetrically with respect to the line connecting s and t, but q is slightly perturbed so that the four nodes a, b, p, and q do not lie on the same circle. Starting from the source node s, the algorithm select $w = a$ or $w = b$ as the next node since $d(s, w) + d(w, t)$ is almost the same as $d(s, t)$, which is a lower bound of the objective function value. We here assume, without loss of generality, that the next node is a. Then, the next node is b since $d(a, b) + d(b, t)$ is almost the same as $d(a, t)$. After that, the current node moves from b to a. In this way, we repeat moving between the two nodes a and b infinitely many times, and cannot reach to the destination t. □

We close this section by showing that DDG condition and SDDG condition are both closed under linear combination with nonnegative coefficients. This implies that the above mentioned algorithms is a basis of a class of the generalized greedy routing algorithm.

Proposition 10. *Suppose that objective functions f and g satisfy* DDG *(resp.* SDDG*) condition. Then, the following objective functions also satisfies* DDG *(resp.* SDDG*) condition:*

- $h_1 = af + bg$ *with any two nonnegative real numbers a and b,*
- $h_2 = f^a g^b$ *with any two nonnegative real numbers a, b, where it is assume that functions f and g take positive numbers, i.e., $f(w, v, t) > 0$ and $g(w, v, t) > 0$ hold for all $w, v, t \in P$,*
- $h_3 = \max\{f, g\}$.

Proof. The proof of the statement for SDDG condition is rather straightforward from (2). Hence, we give a proof of the statement for DDG condition; below we consider the function h_1 only since the proofs for h_2 and h_3 can be done similarly.

Assume, to the contrary, that h_1 does not satisfy DDG condition. Then, there exist distinct nodes $w, v, t \in P$ such that $d(v, t) \leq d(w, t)$ and

$$h_1(w, v, t) \leq \max\{h_1(u, v, t) \mid u \in (P \setminus \{v, t\}) \cap \mathrm{int}D(v, t)\}. \quad (3)$$

Since f and g satisfy DDG condition, it holds that

$$f(w, v, t) > \max\{f(u, v, t) \mid u \in (P \setminus \{v, t\}) \cap \mathrm{int}D(v, t)\},$$
$$g(w, v, t) > \max\{g(u, v, t) \mid u \in (P \setminus \{v, t\}) \cap \mathrm{int}D(v, t)\}.$$

Hence, it follows that

$$\begin{aligned}
&h_1(w, v, t) \\
&= af(w, v, t) + bg(w, v, t) \\
&> a \max\{f(u, v, t) \mid u \in (P \setminus \{v, t\}) \cap \mathrm{int}D(v, t)\} \\
&+ \ b \max\{g(u, v, t) \mid u \in (P \setminus \{v, t\}) \cap \mathrm{int}D(v, t)\} \\
&\geq \max\{af(u, v, t) + bg(u, v, t) \mid u \in (P \setminus \{v, t\}) \cap \mathrm{int}D(v, t)\} \\
&= \max\{h_1(u, v, t) \mid u \in (P \setminus \{v, t\}) \cap \mathrm{int}D(v, t)\},
\end{aligned}$$

which is a contradiction to (3). □

5. Discussion on Designing Ad Hoc Networks

Based on the unified view to greedy geometric routing algorithms obtained in this paper, we discuss how to design efficient ad hoc networks. Listed below are basic properties required for an ad hoc network G (and a geometric routing algorithm on G):

- Connectivity: network G should be connected.
- Sparsity: network G should be sparse.
- Flexibility: network G can be dynamically maintained according to changes of locations/status of nodes of P.
- Long-edge avoidance: use of long edges should be avoided as much as possible.
- Fast transmission: each packet should be sent in a small number of hops.

Connectivity is necessary for the guaranteed delivery of packets in geometric routing algorithms. Sparsity is also necessary to reduce the interference of the transmission among nodes.

Flexibility is particularly important if we consider a mobile ad hoc network where each node may move. In a mobile ad hoc network, its topology dynamically changes when some node enters or leaves the transmission disk of another node. In such a case, the node must broadcast the information of the updated topology to its neighbors; moreover, it may need to broadcast the information globally by the operation called flooding. Such broadcast is expensive and should be avoided as much as possible.

A long edge in an ad hoc network corresponds to a communication using large transmission radius. Hence, use of long edges should be avoided since it increases the power consumption at a node; the amount of power consumption is assumed to be proportional to the cube of the transmission radius in popular models of ad hoc networks. Moreover, a transmission with large radius often causes the interference of communication among nodes. In a mobile ad hoc network, the length of an edge may become longer than the largest possible transmission radius; in such a case the edge disappears from the network and the network topology changes.

On the other hand, use of long edges is unavoidable in general since otherwise the network can be disconnected. Even if a network is connected, greedy geometric routing algorithms often fail to find a path with short edges. For example, Greedy Routing gets stuck if the current node v has no node in its transmission disk $D(v)$ which is closer to the destination t than v, even if there exists a detour path from v to t using short edges. We may design a hybrid method with a routine to find such a detour path (see, e.g., [5], [9]) since it is advantageous to use short edges in a network as much as possible.

Unit disk graph and Gabriel graph are popular graphs often used as ad hoc networks in the literature. Unit disk graph is a sparse graph which can be computed locally, and does not have long edges by definition. On the other hand, unit disk graph is often disconnected; moreover, there are examples of *connected* unit disk graphs for which Greedy Routing and other greedy geometric routing algorithms do not work. Gabriel graph is also a sparse graph which can be computed locally. It is known that a Gabriel graph is connected and contains the Euclidean minimum spanning tree as its subgraph. Greedy geometric routing algorithms on Gabriel graph has been investigated (see, e.g., [5]), although greedy geometric routing algorithms do not have theoretical guarantee of delivery on supergraphs of Gabriel graphs.

In contrast, Delaunay graph enjoys various nice properties: it satisfies connectivity and sparsity requirements, and most of its edges are short. In addition, Delaunay graph can be also computed locally in a distributed fashion. We have shown that Generalized Greedy Routing works on Delaunay graphs and their supergraphs if the objective function satisfies DDG condition. Nevertheless, Delaunay graphs may contain long edges, which should be avoided as much as possible. Another demerit on Delaunay graph is that the maintenance of its topology is often expensive under the setting where nodes may move [7].

As we have seen in Section 4, all of greedy geometric routing algorithms considered in this paper, except for Compass Routing, guarantee packet delivery on any supergraphs of Delaunay graphs. This property is also advantageous from the viewpoint of flexibility since we need not maitain the structure of Delaunay graph exactly if we apply one of these algorithms. We can easily compute the minimum value of the objective function f within the Gabriel disk $D(v, t)$, and send a packet to a neighbor of v with a lower value of f in the currently maintained network (i.e., an old version of Delaunay graph). If there is no such neighbor, then we check the exact positions of nodes within the transmission disk, which can be enlarged if it is necessary to have a neighbor, and update the topology of the graph only locally.

6. Comparison of Greedy Geometric Routing Algorithms

We compare the greedy geometric routing algorithms considered in Section 3 from the viewpoint of fast transmission and long-edge avoidance, which are in general incompatible requirements.

We have observed that the objective functions used in Midpoint Routing, Rainbow, Anchored Wavefront, and Backward Anchored Wavefront satisfy SDDG condition. This means that if G is a supergraph of the Delaunay graph, then each of the algorithms always finds a node in the Gabriel disk, to which a packet is sent. This is advantageous in reducing the possibility that a path becomes highly zigzag. Other algorithms do not have this property since the contour curves may properly intersect the boundary of the Gabriel disk.

We then examine algorithms one by one. In order to reduce the number of hops in the network, it is advantageous to reduce the distance toward the destination as much as possible. For the purpose, Greedy Routing is apparently the best, although it tends to select long edges, and may visit a node outside the Gabriel disk as we mentioned in Section 5.

Midpoint Routing selects shorter edges than Greedy Routing, while keeping the increase in the number of hops small. In addition, Midpoint Routing always selects a node in the Gabriel disk. Midpoint Routing, however, still tends to select a long edge since the midpoint of the current node v and the destination t has the smallest objective function value. Modified Midpoint Routing uses a parameter λ to control this balance, but it uses longer edge than Midpoint Routing, due to the constraint $\lambda \geq 1/2$. Note that the objective function of Modified Midpoint Routing does not satisfy DDG condition if $\lambda < 1/2$, which implies that packet delivery is not guaranteed in such a case.

By definition, Rainbow tends to select a path which is as straight to the destination t as possible since a contour curve closer to the line vt has a smaller value of the objective function. As a side effect of this, the total length of the path is expected to be short in most cases. The algorithm, however, does not control the length of chosen edges in the used path.

Anchored Wavefront and Backward Anchored Wavefront have contour curves which are circles going through v and t, respectively. In this sense, these algorithms resemble Modified Midpoint Routing; in addition, these algorithms enjoy some nice properties. Intuitively, Anchored Wavefront tends to select shorter edges, while Backward Anchored Wavefront selects longer edges with smaller visual angles than Modified Midpoint Routing.

In summary, we should use an appropriate method according to the requirements of an ad hoc network, which are different depending on applications. Our greedy routing algorithms proposed in this paper have nice properties that seem to be desired in many occasions. In addition, we can easily design hybrid version of greedy routing algorithms in response to request of users since DDG condition is closed under linear combination, as mentioned in Proposition 10.

6.1 Experimental Evaluation of Algorithms

Below we evaluate greedy routing algorithms considered in this paper by computational experiments. The criteria of the evaluation are as follows:

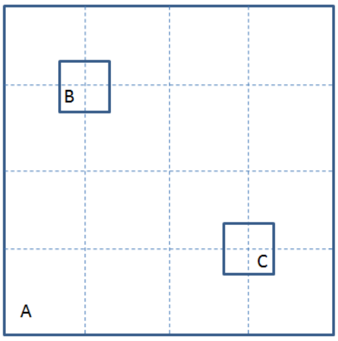

Fig. 7 Experiment setup.

- percentage of success in packet delivery,
- transmission time, which is given by the number of hops,
- transmission power, which is given by the sum of cubes of edge lengths in the path found by the algorithm.

The input of the experiment is given by randomly chosen nodes in the square A whose size is 200×200 (see Figure 7). The source and the destination are random nodes chosen in the squares B and C of size 20×20, respectively. In the experiments, we use the Delaunay graph G of a given set of nodes and its supergraphs. For a parameter $\alpha \in \mathbb{R}_+$, we denote by $G(\alpha)$ the supergraph of the Delaunay graph G obtained by connecting pairs of nodes x, y in G satisfying $d(x, y) \leq \alpha$. Note that $G(0) = G$ holds. We generate 100 inputs for each experiment, and the results shown below is the average numbers taken over the 100 inputs.

Our motivation of using supergraphs $G(\alpha)$ in the experiment is to consider a more "natural" setting than using the origianl Delaunay graph. Indeed, it is possible that some two nodes in a Delaunay graph are not connected by an edge even if they are close to each other, while it is natural that such a pair of nodes can communicate to each other in an ad hoc network. This observation motivates us to consider a supergraph $G(\alpha)$ of a Delaunay graph obtained by adding edges connecting close pairs of nodes.

(1) Experiment 1

We first evaluate the greedy routing algorithms in Section 3, except for Modified Midpoint Routing. The graphs used in the experiment are Delaunay graph G of randomly generated 1,000 nodes, and its supergraphs $G^* = G(13)$ and $G^{**} = G(20)$. Results of experiments are shown in Figure 8.

In Experiment 1, all routing algorithms succeed to deliver packets to destination. Note that Compass Routing alone does not have theoretical guarantee to deliver packets to destination on supergraphs of Delaunay graphs.

It is easily seen from Figure 8 that the performance of Anchored Wavefront with respect to the transmission time and power is almost the same for all three graphs G, G^*, and G^{**}. It is due to the fact that Anchored Wavefront always chooses a close neighbor of the current node as the next node. Hence, even if the algorithm is applied to the su-

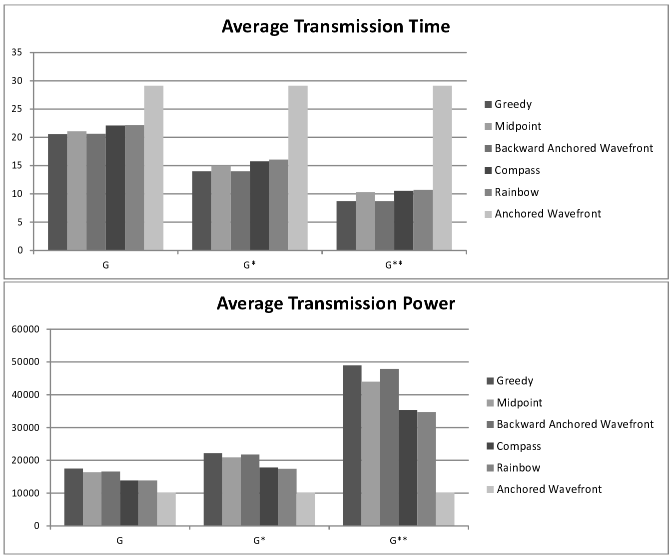

Fig. 8 Average transmission time and power in Experiment 1

pergraphs G^* and G^{**}, it chooses the same node as in G. In addition, Anchored Wavefront requires much larger transmission time and much smaller power than the other algorithms; it takes more than 2.5 times of transmission time and less than about 0.2–0.3 times of transmission power required by other routings algorithms. Hence, Anchored Wavefront is suitable for applications that require small transmission power to deliver packets to destination.

The behavior of the other five algorithms are almost similar; among them, the results of Compass Routing and Rainbow are quite similar and have slightly larger transmission time and smaller transmission power than the other three algorithms. Hence, these algorithms are suitable for application requiring fast transmission of packets. The selection of appropriate methods depends on the requirement of applications and merits and demerits of the algorithms. For example, the four algorithms other than Compass Routing have theoretical guarantee of packet delivery on supergraphs of Delaunay graphs, while Compass Routing uses the angle between two nodes, which is suitable in sensor networks.

(2) Experiment 2

As shown in Proposition 10, DDG and SDDG conditions are closed under linear combination of objective functions. The purpose of Experiment 2 is to evaluate various convex combinations of Greedy Routing and Anchored Wavefront.

In the experiment we use the objective function given by $g(w, v, t) = \beta \times f_G(w, v, t) + f_2(w, v, t)$ with a positive real parameter β; recall that f_G and f_2 are objective functions of Greedy Routing and Anchored Wavefront, respectively. The value β is set to 0, 0.22, 0.35, 0.48, and $+\infty$; $\beta = 0$ and $\beta = +\infty$ correspond to Anchored Wavefront and Greedy Routing, respectively. The graphs we use for experiment are Delaunay graph G of 1000 randomly generated nodes and its supergraphs $G^* = G(10)$, $G^{**} = G(15)$, and $G^{***} = G(20)$.

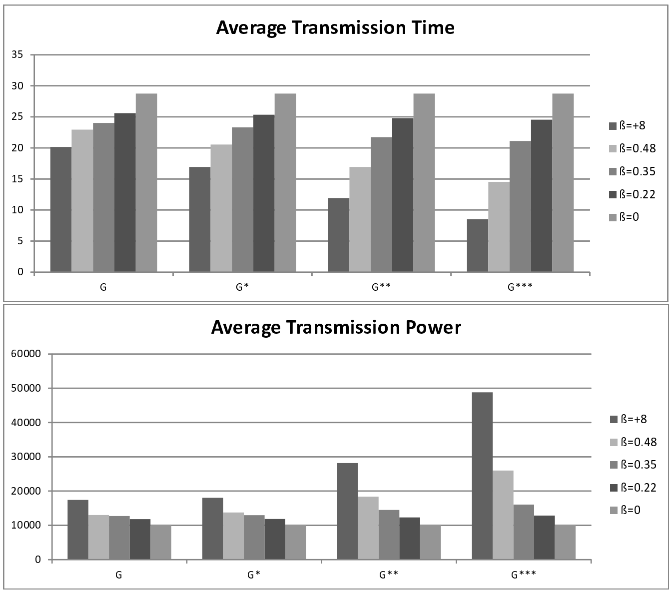

Fig. 9 Average transmission time and power in Experiment 2

Results of experiments are shown in Figure 9.

In Experiment 2, all routing algorithms succeed to deliver packets to destination. If we use a smaller β value, then the combined routing algorithm becomes closer to Anchored Wavefront, which implies that its transmission time becomes larger and transmission power becomes smaller. This experiment shows the tradeoff between transmission time and transmission power by controlling the parameter β, where the "best" choice of β depends on the requirement of applications.

7. Concluding Remarks

The main aim of this paper is to give a unified view to a family of greedy geometric routing algorithms proposed in the literature. In particular, we showed that the guarantee of packet delivery on Delaunay graphs is obtained by a property of the objective function called DDG condition. Based on this result, we proposed new variants of greedy routing algorithms which also have theoretical guarantee of packet delivery on Delaunay graphs. We evaluated the performance of the greedy routing algorithms considered in this paper also by computational experiments.

While we considered geometric routing algorithms in ad hoc networks in Euclidean plane, its three dimensional variants can be naturally considered. In the design of ad hoc networks in real application, however, we often need to consider a metric space with nonuniform distance due to the existence of obstacles and other natural/social conditions. Handling such metric spaces is a challenging task as a future work.

As mentioned before, the objective function used in Compass Routing does not satisfy DDG condition and therefore the algorithm does not always work on supergraphs of Delaunay graphs, while it works on Delaunay graphs. It is an annoying behavior that addition of new edges to Delaunay graphs affects the success of packet delivery by a routing algorithm; note that identifying such additional edges in a network is often expensive in mobile ad hoc networks. Nevertheless, Compass Routing is advantageous on a sensor (e.g., a vision sensor) network since the visual angle between two nodes is easy to measure. Therefore, it is valuable to modify Compass Routing to overcome above mentioned defect by using our unified view. A routing algorithm by Sato and Tokuyama [9], which is briefly explained below, can be seen as such a modification.

Suppose that the destination node t is not within the transmission disk $D(v)$ of v. In such a situation we need to send the packet to a node w contained in $D(v)$. Let D' be the disk around t with radius $d(v, t)$. If the set $L = D(v) \cap D'$ contains a node, then it is better to choose such a node as the next node w. We may choose a node $w \in D(v)$ with $\angle tvw \leq \pi/6$ as the next node since this condition implies $w \in L$. Such an idea is used in the algorithm by Sato and Tokuyama [9]. More precisely, their algorithm uses a graph called the generalized local neighbor graph, which has a directed edge from a node v to a randomly selected node in each sextant of the transmission disk $D(v)$ to give theoretical guarantee on the number of hops. Further investigation in this direction is also needed.

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