Erratum to "Approximability of the Subset Sum Reconfiguration Problem*"

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Abstract. There is a flaw in the proof of Theorem 1 of the original article [2] which claims that the SUBSET SUM RECONFIGURATION problem is strongly NP-hard. This erratum proves that the problem is NP-hard.

1 New Complexity Result

We replace Theorem 1 of [2] with the following theorem.

Theorem 1. SUBSET SUM RECONFIGURATION *is NP-hard.*

Proof. We give a polynomial-time reduction from the PARTITION problem [1] to our problem. In PARTITION, we are given a set U of $n-1$ elements $u_1, u_2, \ldots, u_{n-1}$; each element $u_i \in U$ has a positive integer size $s(u_i)$. Then, the PARTITION problem is to find a subset U' of U such that $\sum_{u \in U'} s(u) = \frac{1}{2} \sum_{u \in U} s(u)$. It is known that PARTITION is NP-complete [1].

Given an instance of PARTITION, we construct the corresponding instance of SUBSET SUM RECONFIGURATION. The set A consists of n items $a_1, a_2, \ldots, a_{n-1}, b$: let $s(a_i) = s(u_i)$ for each i, $1 \le i \le n-1$, and let $s(b) = \frac{1}{2} \sum_{u \in U} s(u)$. Then, each item a_i corresponds to the element u_i in U. The knapsack is of capacity $c = \sum_{u \in U} s(u)$, and set the threshold $k = \frac{1}{2} \sum_{u \in U} s(u)$. Finally, the two packings A_0 and A_t are defined as follows: $A_0 = \{b\}$ and $A_t = U$, and hence both A_0 and A_t are of total size at least k. This completes the construction of the corresponding instance.

We first show that $\mathrm{OPT}(A_0, A_t) \ge k$ if there exists a desired subset U' for the instance of PARTITION. Because $c - s(b) = \frac{1}{2} \sum_{u \in U} s(u) = \sum_{u \in U'} s(u)$, there exists a reconfiguration sequence $\mathcal{P}$ between the two packings A_0 and A_t, as follows: add the items in U' one by one; remove the item b; and add the items in $U \setminus U'$ one by one. Because the removal is executed only for the item b in the reconfiguration sequence $\mathcal{P}$ above, the objective value of $\mathcal{P}$ is $f(\mathcal{P}) = s(U') = \frac{1}{2} \sum_{u \in U} s(u) = k$. Therefore, we have $\mathrm{OPT}(A_0, A_t) \ge f(\mathcal{P}) = k$ if there exists a desired subset U' for the instance of PARTITION.

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Conversely, we show that there exists a desired subset U' for the instance of PARTITION if $\mathrm{OPT}(A_0, A_t) \geq k$. Consider an arbitrary optimal reconfiguration sequence $\mathcal{P}^* = \langle A_0, A_1^*, A_2^*, \ldots, A_{t-1}^*, A_t \rangle$ between the two packings A_0 and A_t. Because $b \in A_0$ and $b \notin A_t$, there exists a packing A_j^* in $\mathcal{P}^*$ which is obtained from A_{j-1}^* by removing the item b. Then,

$$s(A_j^*) = s(A_{j-1}^*) - s(b) \leq c - s(b) = \frac{1}{2} \sum_{u \in U} s(u) = k.$$

On the other hand, $s(A_j^*) \geq f(\mathcal{P}^*) = \mathrm{OPT}(A_0, A_t) \geq k$, and hence $s(A_j^*) = k = \frac{1}{2} \sum_{u \in U} s(u)$. Because $b \notin A_j^*$, we have $A_j^* \subset U$. Therefore, there exists a subset $U' = A_j^*$ of U such that $\sum_{u \in U'} s(u) = \frac{1}{2} \sum_{u \in U} s(u)$ if $\mathrm{OPT}(A_0, A_t) \geq k$.

This completes the proof of the theorem. □

Theorem 1 of this erratum immediately implies the following corollary, which is the replacement of Corollary 1 of [2].

Corollary 1. MAXMIN SUBSET SUM RECONFIGURATION *is NP-hard.*

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References

1. M. R. Garey and D. S. Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness, Freeman, San Francisco, CA, 1979.
2. T. Ito and E. D. Demaine, Approximability of the subset sum reconfiguration problem, To appear in J. Combinatorial Optimization, `DOI:10.1007/s10878-012-9562-z`