Design and Analysis of Information Systems
Topics in Advanced Theoretical Computer Science

Autumn-Winter 2011

Purpose of the lecture

- Design of information systems
  - Statistics
  - Database management and query
  - Internet search
  - Pattern recognition
  - Logic problems
  - Computer graphics
- Mathematical Modeling and Algorithm Design
  - ART and also FUN!

Text Books

- Programming Pearls
  - Jon Bentley 2009 (2nd edition), Addison-Wesley $40
- The art of computer programming vol 1-3
  - (Donald Knuth) 1997 (3rd edition), Addison-Wesley
- Mathematical Puzzles
  - Peter Winkler, 2004, A.K. Peters Ltd.
- The Art of Mathematics
  - Bela Bollobas, 2006, Cambridge Press,

Day 1

Searching, Searching, and Searching

Aha Algorithms (from Programming Pearls)

- I am thinking of a natural number in [1,1000]
  - I write down on a paper
- You must find it
  - You can ask questions (question should be short)
  - I answer yes or no
- Your task: write down your strategy
- Winner: use least number of questions

Finding missing number

- We have different 32bit numbers in a sequential array of length 4000,000,000
  - Find a missing number, using only one-word memory
    - One word can contain 64 bits
  - If you are allowed to use 1M byte memory, what you will do?
- We have 50 cards among 52
  - In one trial, all cards are shown in a random order
  - Find a missing card with least number of trials

Debugging a program

- I have a computer program with 1000 lines of codes, but it has a bug in a line, and aborts.
- Find out the bug
  - You can place a stopper at any line and run the program. The program runs correctly if there is no bug until then.

Computing power

- Write a fast code to compute the t-th power of a given natural number n.
- Write a fast code to compute the t-th power of n modulo p.
  - If you know p is prime, and p is much smaller than t, can you do better? (say, p = 31 and t = 10000)

Searching in the internet

- We have 1,000,000 documents each contains 10,000 characters (in alphabet).
  - Given any keyword (say, “Tokuyama”), find all documents containing it
  - Given any string (say, “keshi Tokuy”), find all documents containing it
- You can **preprocess** the data
  - What is preprocess? Make the data well-ordered so that we can find necessary information easily.

We may encounter hard problems..

- Does the following program terminates for every integer x ? (Bentley, programming pearls, exercise of Chap 4)

  while x > 1 do

  if even(n) then x = x/2

  else x = 3x + 1

- If x = 10,
  - 10 →5→16→8→4→2→1
- If x = 9,
  - 9→28→14→7→22→11→34→17→52→26→13→40→20→10→

Hint by Jon Bentley: If you solve this problem, run to the nearest mathematics department and ask for a Ph. D.

Engineers should be flexible

- An exercise in “Programming Pearls”
  - NASA needed for writing implements that work well in the extreme environment of space. A million dollar project research developed a “space pen”. According to the legend, how did the Soviets solve the same problem?
- I could not imagine the solution, but probably you can.

QUIZES? ALGORITHMS!

Exchange two strings

- Given an array of length n = ab, which is concatenation of two strings a and b
- Change the array to n’ = ba, but we can only use additional one word memory
  - a = 0302040552, b = 02035530219852
  - n = 030204055202035530219852
  - n’ =020355302198520302040552
- Write a smart program

USE OF RANDOMNES

Data checking

- We have n data {1,2,…,n} in a random order
  - n is huge, and read from external disk
  - We suspect that one data is modified by the enemy. Can you check it by using single word additional memory?
- Can we check if we do not know how many data are modified?

Use of randomness

- Given 10000 nuts and 10000 bolts forming 10000 matching pairs. We can compare a pair of bolts (or nuts) and see which is larger. How to find one matching pair quickly?

Birthday trick

- If we have 20 students in a class, what is the probability that there is a pair of students with the same birthday?

Danger of randomness

- Lets make a deal! (host: Monty Hall)
  - There are three doors , behind one of which contains a gorgeous car.
  - You pick one of them
  - The host comedian opens one of the unselected doors, which is open.
  - Now, “change or remain”.
- Too easy? Then….

Marilyn vos Savant

INVARIANTS

Generalized Tick Tack Toe

- Let $X = \{S1, S2, \ldots S7\}$ where each Si is a set with 3 elements (some elements may be shared by some sets)
- Alice and Bob color elements in turn. Alice colors red, and Bob colors blue (each element can be colored only once).
- If Alice makes a set with all red elements, Alice wins, otherwise Bob wins.
- If Bob starts, prove that Bob always wins.
- If Alice starts, show that Alice may win (it depends on X)

Black and white beans

- We have 25 white beans and 20 black beans in a bin.
- We pick two beans randomly.
  - If they are same color, remove them, and throw in an extra black bean.
  - If they are same colors, return the white bean, and remove the black bean
- What is the probability that the last remaining bean is white?

Clock Solitaire game

- We divide 52 playing cards into 13 card decks, each contains 4 cards. We number the deck to 1 to 13.
- Starting the deck of number 13 we open cards, where we next open the deck indicated by the card.
  - E.g., if we find 4 of harts, we next open the deck 4.
  - If the deck is empty, we loose
- We win if all cards are open
- What is the probability of win.